
International Experience on Longevity Risk

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Agenda

- Introduction
 - Development of Mortality Tables
 - Capital Market Solutions
 - Comments
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Introduction

- Should the demand for individual annuity products increase in the future?
- US: Social security reform(DB→DC)
Taiwan: 勞退新制, 退休金所得替代率仍偏低



Development of Mortality Tables

- Incorporating the Trend of Mortality Improvement.
 - Projected Mortality=Base Mortality* Reduced Factor
 - Example: UK, German
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Development of Mortality Tables(Con't)

age/year		t	t+1	t+2	t+3	
x		$q_x(t)$	$q_x(t+1)$	$q_x(t+2)$	$q_x(t+3)$	
x+1		$q_{x+1}(t)$	$q_{x+1}(t+1)$	$q_{x+1}(t+2)$	$q_{x+1}(t+3)$	
x+2		$q_{x+2}(t)$	$q_{x+2}(t+1)$	$q_{x+2}(t+2)$	$q_{x+2}(t+3)$	
x+3		$q_{x+3}(t)$	$q_{x+3}(t+1)$	$q_{x+3}(t+2)$	$q_{x+3}(t+3)$	

Development of Mortality Tables(Con't)

- a(55):
 - this table was produced in about 1950, and designed to give reasonable values of annuities purchased in 1955 (purchased immediate annuities). This table is applied to both males and females. (CMIR 1, 1973.)
- PA90
 - this table is based on the experience of pensioner annuities from 1967 to 1970, and published by the CMIB in 1978. (CMIR 3, 1978.)
- **PMA80 Base**
 - Pensioners, Male, Amounts, based on the experience from 1979 to 1982. Forecast factors allow forecasting of values for any age to any calendar year and year of birth tables applied too. This table was published in 1990. (CMIR 9, 1988 and CMIR 10, 1990.)
- **PMA92 Base**
 - Pensioners, Male, Amounts, based the experience from 1991 to 1994. This table has the same form as the 80 series with different forecast factors, and was published in 1998. (CMIR 16, 1998 and CMIR 17, 1999.)

Development of Mortality Tables(Con't)- UK

- GMIB and GAD

- Base Table [q_x]

→ Forecasting mortality rates [$q_{x,t} = q_x * RF_{x,t}$]

→ So far, it is still deterministic.

Example: PMA80 Table

- Calculating μ_x : in contrast to the PMA80 base table, a single formula for μ_x was used for all ages.

$$\mu_x = GM(r, s)(t)$$

$$GM(2, 3)(t) = a_1 + a_2 t + \exp\left[b_1 + b_2 + b_3(2t^2 - 1)\right]$$

$$100a_1 = 0.023$$

$$100a_2 = -0.011$$

$$b_1 = -5.397782$$

$$b_2 = 6.622746$$

$$b_3 = -1.6.$$

- Projecting future $q_{x,t}$ from q_x

$$q_{x,t} = q_x \cdot R F_{x,t}$$

$$R F_{x,t} = \alpha(x) + [1 - \alpha(x)] \cdot [1 - f(x)]^{t/20}$$

where

$$\alpha(x) = \begin{cases} c & x \leq 60 \\ 1 + (1 - c) \frac{x - 60}{50} & 60 \leq x \leq 110 \\ 1 & x \geq 110 \end{cases}$$

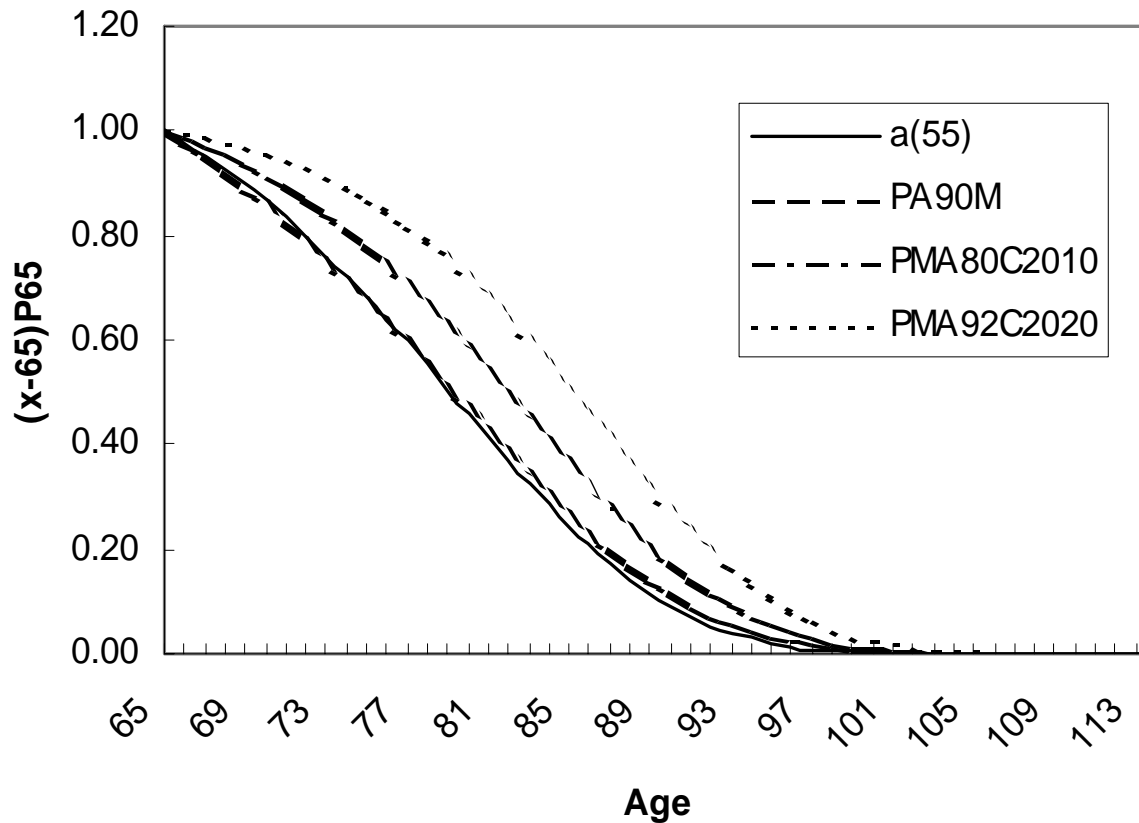
and

$$f(x) = \begin{cases} h & x \leq 60 \\ \frac{(110 - x)h + (x - 60)k}{50} & 60 \leq x \leq 110 \\ k & x \geq 110 \end{cases}$$

where $c = 0.13$ and $k = 0.55$

Development of Mortality Tables(Con't)- UK:

The trend of the survival probability of British people aged above 65 for different mortality tables



Development of Mortality Tables(Con't)- German

- Annuity tables: mortality rates per birth year
+ a trend assumption relating to
the future mortality improvements

 - Published Life Annuity Table
 - DAV1994 R,
 - **DAV 2004 R (Data : Between 1995 and 2002)**
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Development of Mortality Tables(Con't)- German

- An age and calendar dependent mortality model was

$$\frac{q_{x,t+1}}{q_{x,t}} = \exp(-F(x,t)).$$

Current Solutions: Actuarial Methods

The trend function $F(x,t)$ may be expressed by the formula

$$F(x,t) = \begin{cases} F_1(x) & 1999 \leq t \leq 1999 + T_1 \\ F_1(x) \cdot \left(1 - \frac{t - 1999 - T_1}{T_2 - T_1}\right) + F_2(x) \cdot \frac{t - 1999 - T_1}{T_2 - T_1} & 1999 + T_1 \leq t \leq 1999 + T_2 \\ F_2(x) & t \geq 1999 + T_2 \end{cases}$$

Parameter combinations such as $(T_1 = 5, T_2 = 10)$, $(T_1 = 10, T_2 = 15)$ or $(T_1 = 15, T_2 = 20)$ may be appropriate.

Development of Mortality Tables(Con't)- International Comparison

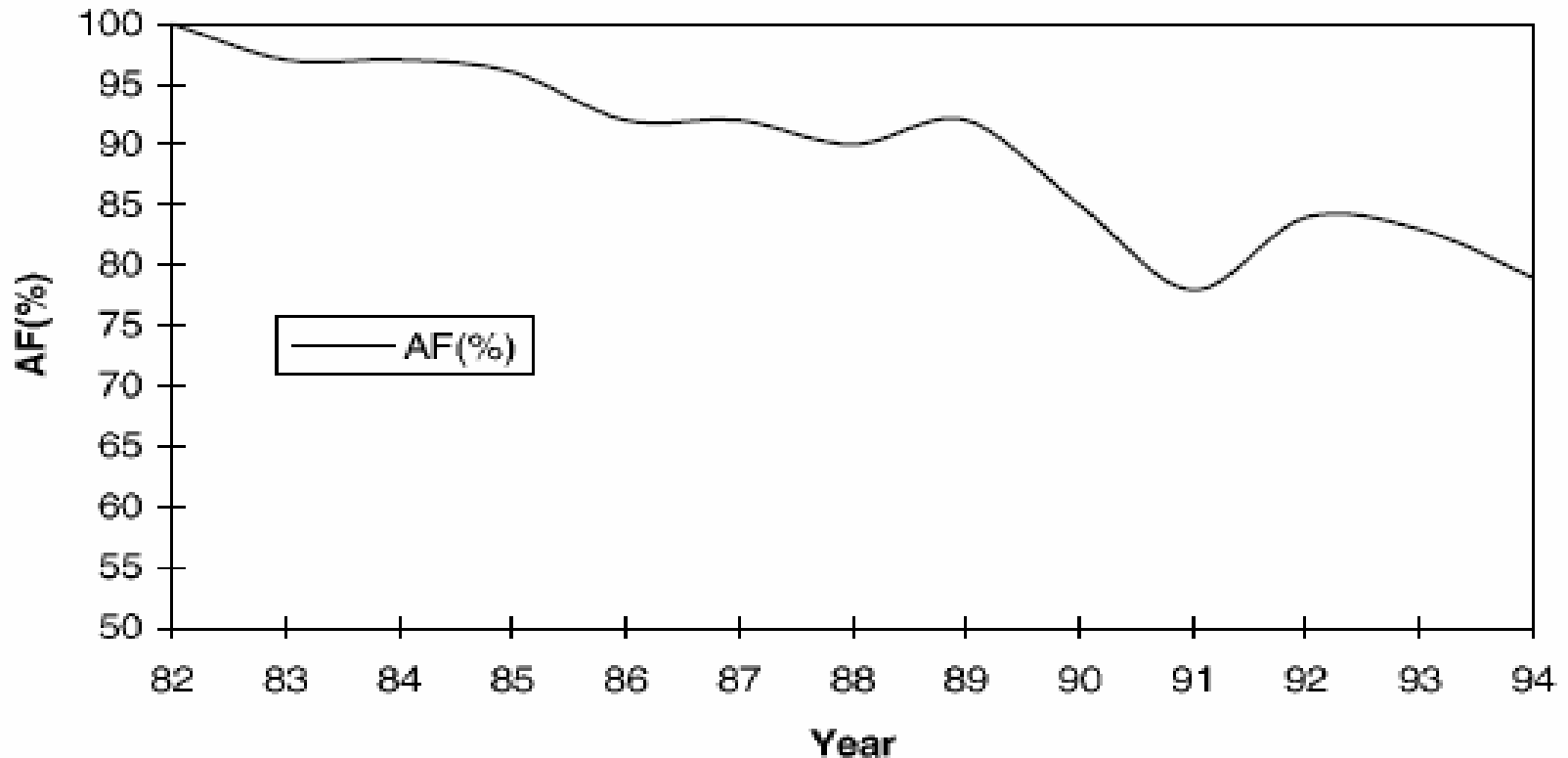
Average for ages 60-89	1960-1979	1965-1984	1970-1989	1975-1994	1980-1999
Males					
West Germany	0.996	0.987	0.984	0.983	0.982
Denmark	0.997	0.997	0.997	0.996	0.992
Japan	0.977	0.974	0.974	0.98	0.984
France (1978-1997)	0.991	0.987	0.984	0.98	0.98
UK (1979-1998)	0.994	0.991	0.987	0.983	0.981
Italy	0.996	0.993	0.988	0.982	0.979
Austria	0.994	0.989	0.984	0.98	0.978
Sweden	0.997	0.995	0.992	0.986	0.983
Switzerland	0.988	0.986	0.985	0.984	0.981
USA	0.992	0.986	0.986	0.987	0.986
Females					
West Germany	0.986	0.979	0.978	0.979	0.98
Denmark	0.982	0.985	0.993	0.998	0.998
Japan	0.972	0.968	0.965	0.967	0.97
France (1978-1997)	0.982	0.979	0.977	0.975	0.976
UK (1979-1998)	0.99	0.99	0.988	0.986	0.985
Italy	0.985	0.983	0.98	0.976	0.975
Austria	0.99	0.984	0.98	0.977	0.975
Sweden	0.981	0.982	0.984	0.984	0.985
Switzerland	0.977	0.974	0.975	0.979	0.98
USA	0.984	0.982	0.986	0.991	0.994

Development of Mortality Tables(Con't)- German

- Note: An analysis of the observation material shows the relationship between **mortality** and **annuity levels**.
→ weighted by annuity levels

Annual annuity amount (euros)	Males	Females
0 – 600	117%	111%
601 – 1200	110%	105%
1201 – 2000	101%	99%
2001 – 3500	90%	88%
3501 – 6000	89%	91%
> 6000	86%	91%
Aggregate	100%	100%

How do these projecting models interpret for UK experience?



Capital Market Solutions

- Securitization of mortality risk
 - Existing Products
 - Swiss Re mortality bond(2003)
 - EIB/ BNP Paribas/ PartnerRe Longevity Bond(2004)
 - Alternatives
 - Product Swap
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Securitization of mortality risk

- 透過資本市場取得承保能量
 - 人壽保險公司將年金險或退休金商品之長壽風險透過證券化的方式，轉到金融市場上分散給投資者購買，進而分散長壽風險

- Advantage
 - 從保險角度看，保險風險證券化將再保險容量從原來保險業內部擴大到了外部的資本市場；從金融角度看，保險風險證券化具體實施過程就是一個資產證券化過程，這裏的資產就是保險公司的負債。

- Disadvantage:
 - 保險風險證券化的透明度較差
 - 保險風險證券化的運作成本較高
 - 違約風險

Securitization of mortality risk:

Products Design

■ **Survivor Bonds**

- where coupon payment are linked to the number of survivors in a given cohort (Blake and Burrows (2001) and Lin and Cox (2004))

■ **Survivor Swaps**

- where counterparties swap a fixed series of payments in return for a series of payments linked to the number of survivors in a given cohort (Dowd *et al* (2004))

■ **Annuity Futures**

- where prices are linked to a specified future market annuity rate.
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Securitization of mortality risk: (con't)

Products Design

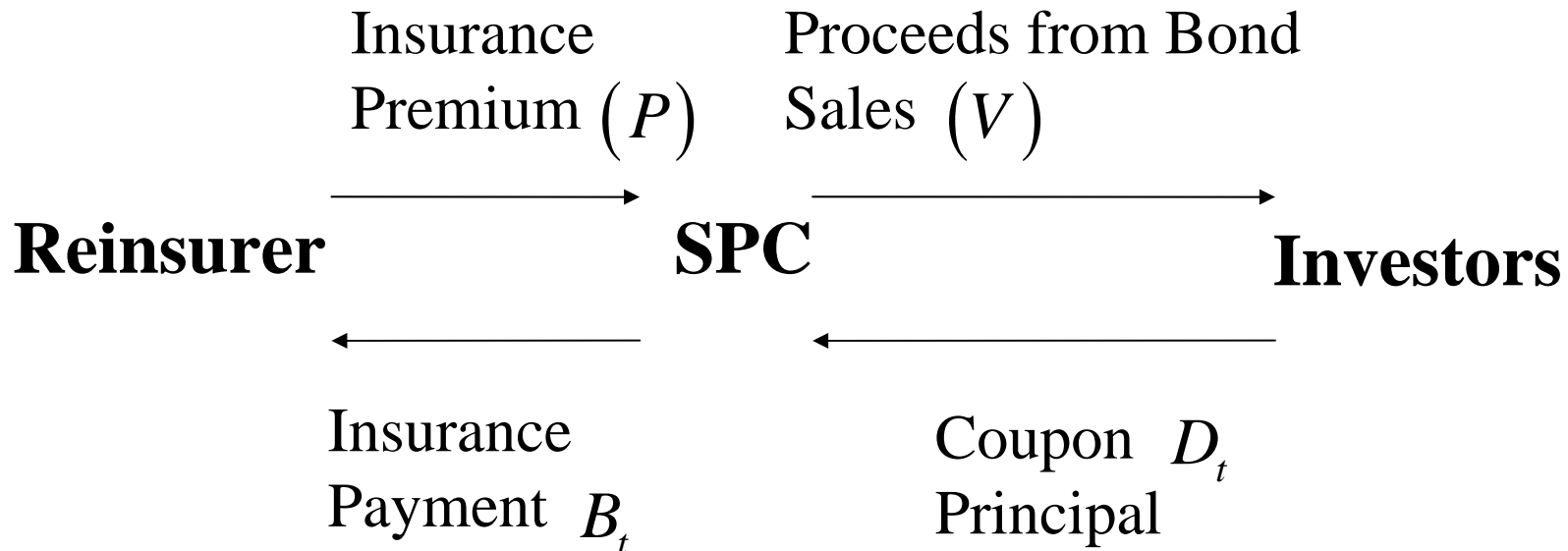
■ **Mortality Options**

- a range of contracts with option characteristics whose payoff depends on an underlying mortality table at the payment date. Example: Guaranteed annuity contract.



Securitization of mortality risk:

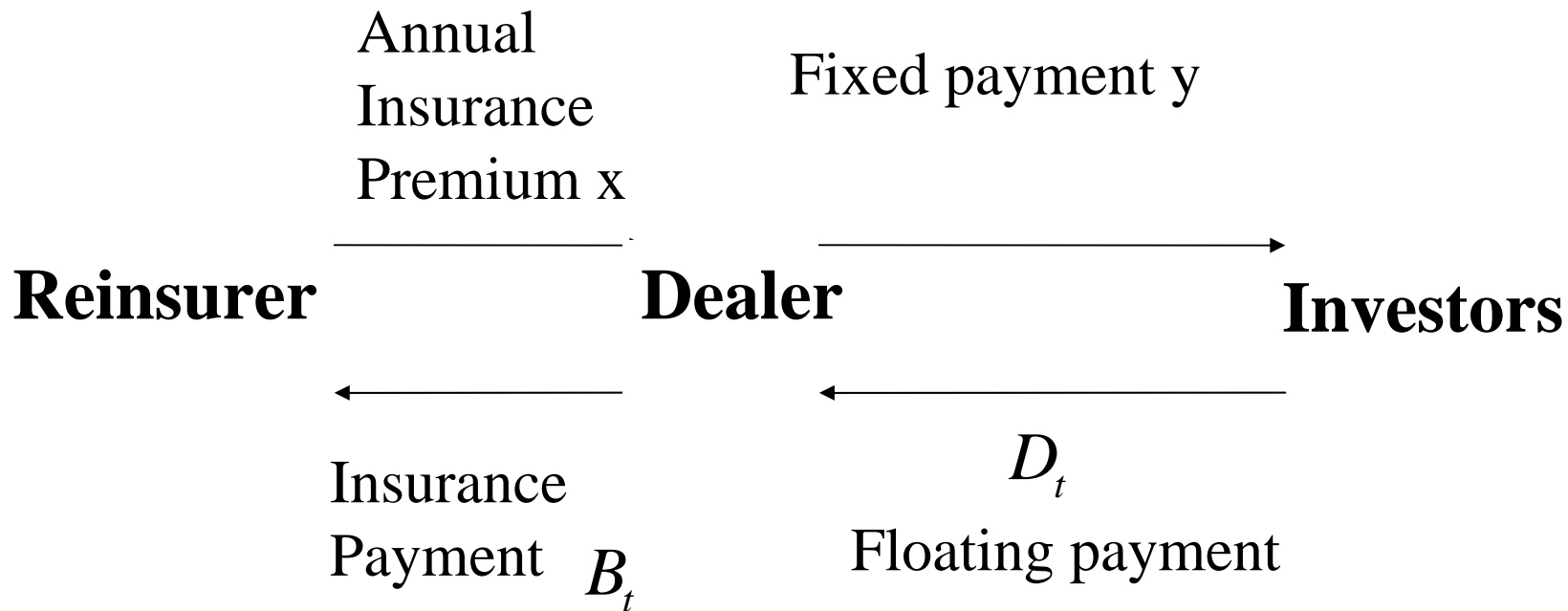
Products Design: an example (Lin and Cox(2004))



$$B_t + D_t = 1000C$$

Securitization of mortality risk:

Products Design: an example (Lin and Cox(2004))



Existing Product

Swiss Re mortality bond

- In December 2003, Swiss Re sponsored a \$400 million securitization of mortality risk.
 - The purpose was to get protection against extreme mortality events
 - → Life Insurance
 - A catastrophe bond structure was used.
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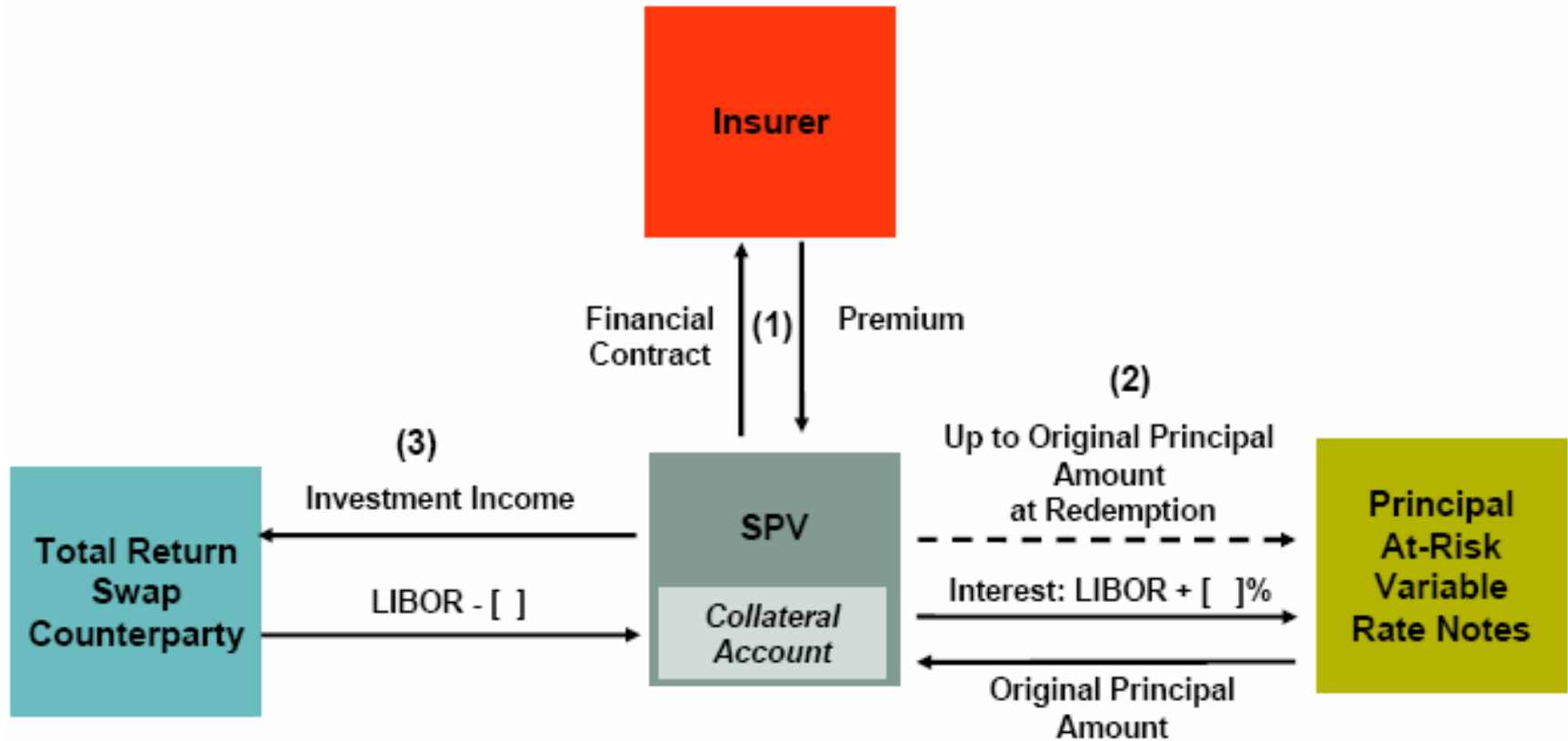
Capital Market Solutions-

Swiss Re mortality bond :Structure

- 保險公司跟SPV簽訂契約, 保險公司交一筆合約金額, 約定SPV外募集一筆資金, 作為保險公司當死亡率過高時所須之超額理賠的部份。
- SPV在外向投資人以票據的方式, 約定到期給付方式, 以死亡率index值為準, 若index超過 $(100+x)\%$ 時投資人之收益減損, 若損失到達 $(100+y)\%$ 時, 投資人就無任何收益。投資人所損失之金額即作為保險公司之超額賠償, 用以彌補保險人因高死亡率所生之理賠損失。若index未達 $(100+x)\%$ 時, SPV依約給付給投資一筆加計利息之最終給付
- 當SPV收到在外募集之資金後, 進行投資, 再向另一金融機構, SWAP以投資收益交換固定金額(內含另一風險), 此作法之目的在於避免投資收益不佳時所生之損失

Capital Market Solutions-

Swiss Re mortality bond : Structure



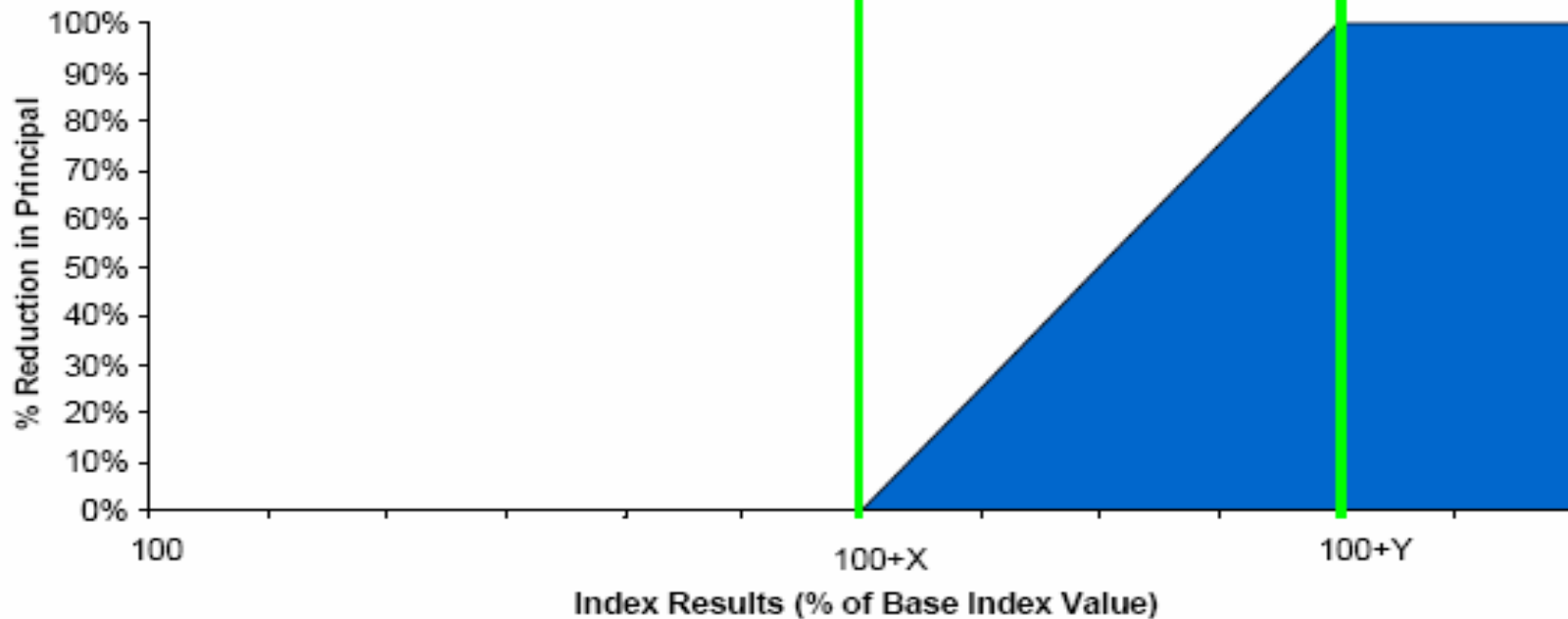
Capital Market Solutions-

Swiss Re mortality bond : Calculation of Loss

$$\% \text{ LOSS} = 100 \times \frac{\text{Index Value} - X}{Y - X}$$

Attachment Point:
[100+x]%

Exhaustion Point:
[100 + y]%



Capital Market Solutions-

Swiss Re mortality bond : Index Value

- Trigger Definition : The index value for a given year is defined to be the average death rate per 100,000 for pre-defined coverage area.

- $$Index = \sum_{j=1} c_j \sum_{i=1} (g^m a_i q_{i,j}^m + g^f a_i q_{i,j}^f)$$

- 1. c_j is the weight for country j
(美國、英國、法國、義大利、瑞士)
- 2. $g^{m,f}$ is the gender weighting
- 3. a_i is the weight for age band i
- 4. $q_{i,j}$ is the observed death per 100,000 for males and females, respectively, from country j and age band i

Capital Market Solutions-

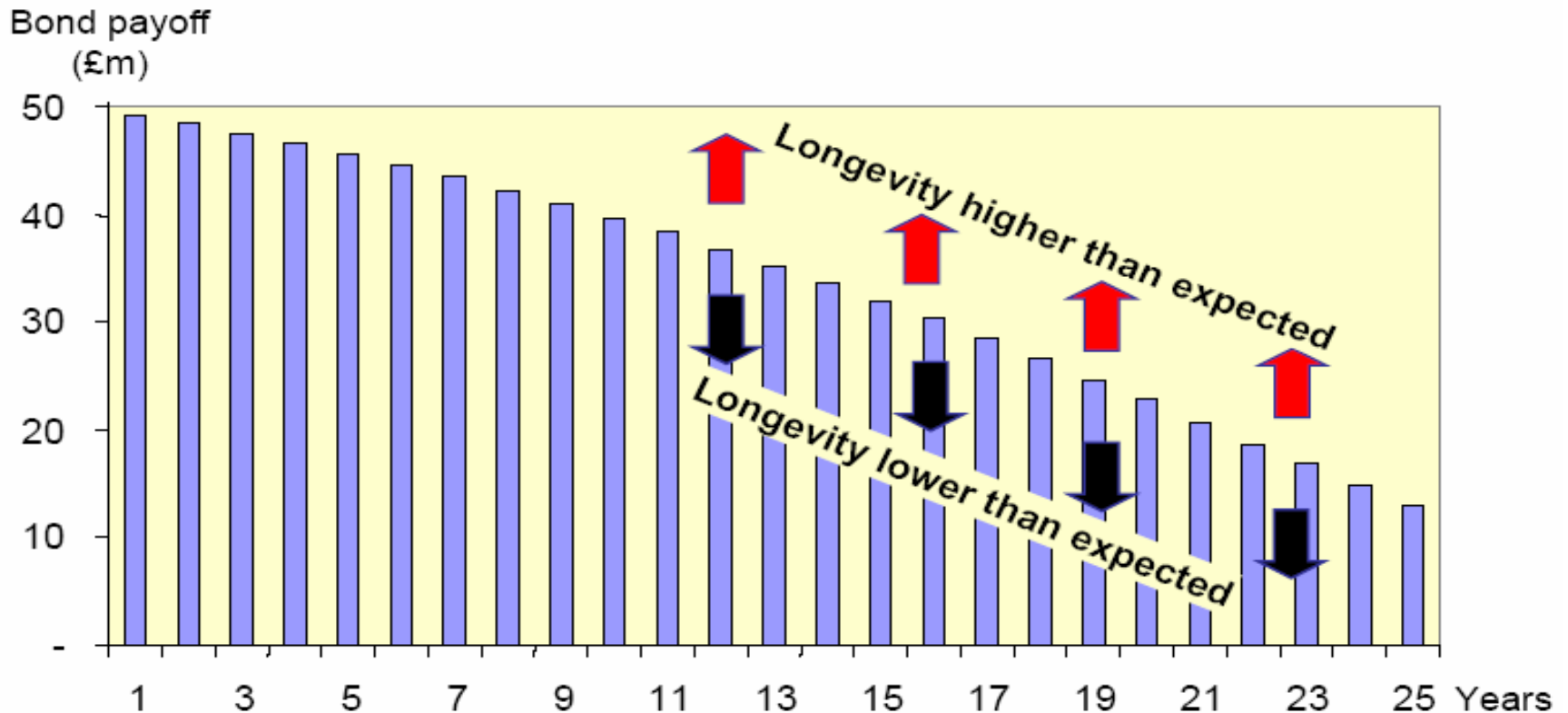
The EIB/ BNP Paribas longevity bond

Issuer	European Investment Bank
Security	£[550]m Longevity-linked EMTN
Index	Based on publicly available ONS data of English and Welsh mortality for a cohort of males aged 65 in 2003
Longevity Risk Period	Calendar years 2003 to 2027 inclusive
Maturity	25 years
Bond payoff	£50,000,000 * CSR_t
CSR_t	Cumulative Survival Rate (i.e. proportion of survivors) in the cohort at time t
Index Published	ONS Publication DH1 Mortality Statistics Table 8
Payment Frequency	Annual
Pricing	T.B.A.
Issue Date	T.B.A.
Calculation Agent	BNP Paribas

Capital Market Solutions-

The EIB/ BNP Paribas longevity bond : **Cash Flow Profile**

Illustrative cash flows of the Longevity Bond based on the latest Government Actuary's Department (GAD) projections



Capital Market Solutions-

The EIB/ BNP Paribas longevity bond: Annual Pay-off

Example based on hypothetical mortality data

Fixing date	2005	2006	2007
Mortality reference year	2003	2004	2005
Age of the cohort	65y	66y	67y
Annual mortality rate	1.50%	1.80%	2.10%
Annual survival rate	98.50%	98.20%	97.90%
Cumulative survival rate	98.50%	96.73%	94.70%
		[98.50% x 98.20%]	[96.73% x 97.90%]
£1,000 Fixed Annuity	£985 Bond pay-off Year 1	£967.3 Bond pay-off Year 2	£947.0 Bond pay-off Year 3

In the current interest rate environment, each £1,000 of Fixed Annuity equates to approximately £11,000 of bond notional

Capital Market Solutions-

The EIB/ BNP Paribas longevity bond : **Advantages**

- The Longevity Bond
 1. It provides a better match for the liabilities of pension funds and life insurers than other available investments other than purchasing (re)insurance to cover the longevity risk.
 2. Life insurers holding the longevity bond as a hedge may be able to hold lower prudential margins. The payoff is similar to a pension in payment.
 - The longevity index is completely transparent and independent - data are published by the UK Government.
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Capital Market Solutions-

The EIB/ BNP Paribas longevity bond :Disadvantages

- Main sources of basis risk:
 - ◆ Different trends in longevity among populations of the same age
 - ◆ Age distribution of the pension fund different from that of the Reference Population. The Reference Population weights all lives equally
 - ◆ Pension fund population includes males and females whereas the Reference Population is male only
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Current Solutions: Financial Methods

The EIB/ BNP Paribas longevity bond: **Disadvantages**

- ❑ A small scheme will find it difficult to use this bond to match its liabilities as the variance between actual and expected mortality will be quite large.
 - ❑ The bond is a progressively worse hedge for pension liabilities related to younger or older cohorts.
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Current Solutions: Financial Methods

Implication of Existing Products

- A market for mortality-based securities will develop if the prices and contracting features make the securities attractive to potential buyers and sellers
 - Revised current product: index
 - Note: **The EIB/ BNP Paribas longevity bond is similar to life annuity issued by insurer.**
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Alternatives: Mortality swaps

